The beam AB is supposed for the moment to have no weight. Consequently the only force acting upon the portion of the beam AO is the reaction R, and, similarly, R' is the only force acting upon the portion OB. Obviously so far as the simple action of these two forces or reactions is concerned, the tendency of each is to cause vertical slices of the beam, so to speak, to slide over each other. In other words, in engineering language, the portion AO of the beam is subjected to the shear $S=R$, while OB is subjected to the shear $S'=-R'$. The cross-sectional area of the beam must be sufficient to resist the shear S or S'. The upper part of Fig. 13 shaded with broken vertical lines indicates this condition of shear. It is evident from this simple case that the total vertical shears at the ends of any beam will be the reactions or supporting forces exerted at those ends, and that each will remain constant for the adjoining portion of the beam.

The third member of equation (13) shows that the greatest bending moment M_1 in the beam varies as the product x_1x_2 of the segments of the span. That product will have its greatest value when $x_1=x_2$. Hence *a simple beam loaded by a single weight will be subjected to the greatest possible bending moment when the weight is placed at the middle of the span, at which point also that moment will be found.*

82. Bending Moments and Shears with any System of Loads.— The general case of a simple beam loaded with any system of weights whatever may be represented in Fig. 13, in which the beam of Fig. 12 is supposed to carry three loads, w_1, w_2, w_3. The spacing of the loads is as shown. The reactions or supporting forces R' are determined precisely as in Fig. 12, each reaction in this case being the resultant of three loads instead of one. Applying the law of the lever as before, the reaction R will have the value

$$R=W_3\frac{d}{l}+W_2\frac{d+c}{l}+W_1\frac{d+c+b}{l}. \quad . \quad . \quad . \quad (14)$$

A similar value may be written for R', but it is probably simpler, after having found one reaction, to write

$$R'=W_1+W_2+W_3-R. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (15)$$

As the beam is supposed to have no weight, no load will act upon the beam between the given weights. The bending moments at the points of application of the three weights or loads will be

$$\left.\begin{aligned} M_1 &= Ra, \\ M_2 &= R(a+b) - W_1 b, \\ M_3 &= R(a+b+c) - W_1(b+c) - W_2 c. \end{aligned}\right\} \quad . \quad . \quad . \quad . \qquad (16)$$

After substituting the value of R from equation (14) in equations (16) the values of the latter are at once known.

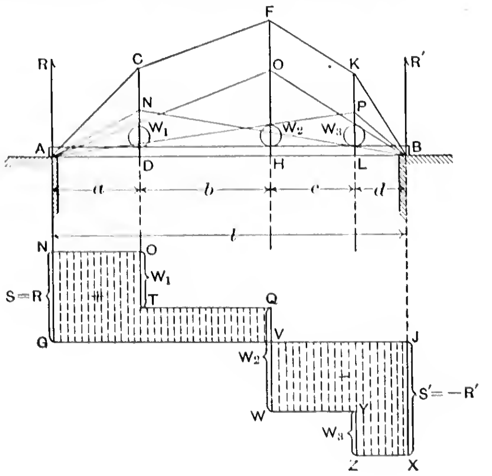


FIG. 13.

The bending produced by each weight will also be represented precisely like that in Fig. 12. The triangle ANB represents the bending produced by W_1; AOB the bending produced by W_2; and APB the bending produced by W_3. The resultant bending effect produced by the three loads or weights acting simultaneously is simply the summation of the three effects each due to a single load. Hence DC is erected vertically through the point of application of W_1, so as to equal DN added to the two vertical intercepts between AB and AP, and AB and AO. Similarly, HF is equal to HO added to the intercepts between AB and AP, and AB and BN. Finally, KL is equal to PL added to the other two intercepts, one between AB and BN, and the other between

AB and BO. Straight lines then are drawn through A, C, F, K, and B. Any vertical intercept between AB and $ACFKB$ will represent the bending moment in the beam at the corresponding point. Obviously any number of loads of any magnitude, or a uniform load, may be treated in precisely the same way.

An important practical rule can readily be deduced from the equations (16), each one of which may be regarded as a general equation of moments. If the system of three, or any other number of loads, be moved a small distance Δx, while they all remain separated by the same distances as before, the bending moment M will be changed by the amount shown in equation (16*a*):

$$\Delta M = R\Delta x - W_1\Delta x - W_2\Delta x - \text{etc.} \quad . \quad . \quad . \quad (16a)$$

If the notation of the differential calculus be used by writing the letter d instead of Δ, and if both members of equation (16*a*) be then divided by dx, equation (16*b*) will result:

$$\frac{\Delta M}{\Delta x} = \frac{dM}{dx} = R - W_1 - W_2 - \text{etc.} = \text{shear.} \quad . \quad (16b)$$

The second member of this equation shows the sum of all the external forces acting on one portion of the beam, that portion being limited by the section about which the moment M acts. That sum of all the external forces, as given by the second member of equation (16*b*), is evidently the total transverse shear at the section considered. Equation (16*b*) then shows, in the language of the differential calculus, that the first derivative of M in respect to x is equal to the total transverse shear. It is further established in the differential calculus that whenever a function, such as M, the bending moment, is a maximum or a minimum, the first derivative is equal to zero. The application of this principle to equation (16*b*) shows that the bending moment in any beam or truss has its greatest value wherever the shear is zero. Hence, in order to determine at what section the bending moment has its greatest value in any loaded beam carrying a given system of loads, it is only necessary to sum up all the forces or loads, including the reaction R, on that beam from one end to the point where that sum or shear is zero; at this latter point the greatest moment sought will be found. This is a very simple method of determining the section at which the greatest moment in the beam exists.

The preceding formulæ and diagrams may be extended to include any number of loads, and they are constantly used in engineering practice, not only for beams and girders in buildings, but also for bridges carrying railroad trains. Whatever may be the number of loads, the expressions for the bending moments at the various points of application of those loads are to be written precisely as indicated in equations (16). When the number of loads becomes great the number of terms in the equations correspondingly increase, but in reality they are just as simple as those for a smaller number of loads.

The diagram for the vertical shear in this beam is the lower part of Fig. 13. As in the case of Fig. 12 the shear at A is the reaction R, as it is R' at the other end of the beam. The shear in the portion AD of the beam has the value R, but in passing the point D to the right the weight W_1 represented by OT must be subtracted from R, so that the shear over the section b of the span is $R-W_1$ or QV in the diagram. Similarly, in passing the point H toward the right, both W_2 and W_1 must be subtracted from R, giving the negative shear (the previous shear being taken positive) VW. The negative shear VW remains constant throughout the distance c, but is increased by W_3 at the point L, so that throughout the distance d the shear $S' = -R'$. These shear values are all shown in the lower portion of Fig. 13 by the vertical shaded lines. Obviously it is a matter of indifference whether the shear above the straight line GJ is made positive or negative; it is only necessary to recognize that the signs are different.

In the case of heavy beams, either built or rolled, as in railroad structures, it is of the greatest importance to determine both the bending moments and the shears, as represented in the preceding equations and diagrams, and to provide sufficient metal to resist them.

The case of Fig. 13 is perfectly general for moments and shears, and the methods developed are applicable to any amount or any system of loading whatever.

83. Bending Moments and Shears with Uniform Loads.—Fig. 14 represents what is really a special case of Fig. 13, in which the loading is uniform for each unit of length of the beam throughout the whole span l. Inasmuch as the load is uniformly distributed, it is evident that the reaction at each end of the beam will be one half the total load, or

$$R = R' = \frac{wl}{2}. \quad \ldots \ldots \ldots \quad (17)$$

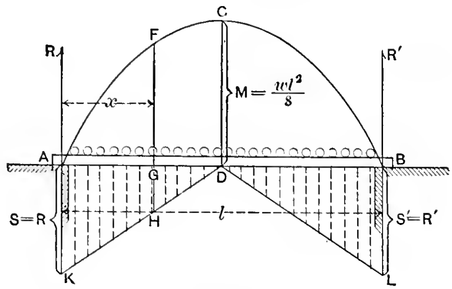


FIG. 14.

The general expression for the bending moment at any point G in the span, and located at the distance x from the end A, will take the form

$$M = Rx - wx.\frac{x}{2} = \frac{w}{2}x(l-x). \quad \ldots \ldots \quad (18)$$

This equation, giving the value of M, is the equation of a parabola with the vertex over the middle of the span. The bending moment at the latter point will be found by placing $x = \frac{l}{2}$ in equation (18), which will give

$$M = \frac{wl^2}{8}. \quad \ldots \ldots \ldots \quad (19)$$

Hence, in Fig. 14, if the vertical line DC be erected at D, so as to represent the value of M in equation (19) to a convenient scale, the parabola ACB may be at once drawn. Any vertical intercept, as GF between AB and the curve $AFCB$, will represent by the same scale the bending moment in the beam at the point indicated by the intercept. Equation (19), giving the greatest external bending moment in a simple beam due to a uniform load, is constantly employed in structural work, and shows that

that moment is equal to the total load multiplied by one eighth of the span.

It has already been shown, in connection with Fig. 12, that when a single centre weight rests on a beam the centre bending moment is equal to that weight multiplied by one fourth the span. If the total uniform load in the one case is equal to the single load in the other, these equations show that the single centre load will produce just double the bending moment due to the same load uniformly distributed over the span. Wherever it is feasible, therefore, the load should be distributed rather than concentrated at the centre of the span.

That portion of Fig. 14 shaded with vertical lines shows the shear existing in the beam. Evidently the shear at each end is equal to the reaction, or one half the total load on the span. The expression for the shear at any point, as G, distant x from A will be

$$S = R - wx = w\left(\frac{l}{2} - x\right). \quad . \quad . \quad . \quad . \quad . \quad (20)$$

If $x = \frac{l}{2}$ in equation (20), S becomes equal to zero. In other words, there is no shear at the centre of the span of a beam uniformly loaded. Hence, if at each end of the span a vertical line AK or BL be laid off downward, and if straight lines KD and DL be drawn, any vertical intercept, as GH, between these lines and AB will represent the shear at the corresponding point. Equation (20) also shows that the shear S at any point is equal to the load resting on the beam between the centre D and that point. Although this case of uniform loading is a special one it finds wide application in practical operations.

84. Greatest Shear for Uniform Moving Load.—The preceding loads have been treated as if they were occupying fixed positions on the beams considered. This is not always the case. Many of the most important problems in connection with the loading of beams and bridges arise under the supposition that the load is movable, like that of a passing railroad train. One of the simplest of these problems, although of much importance, consists in finding the location of a uniform moving load, like that of a train of cars, which will produce the greatest shear at a given point of a simple beam, such as that represented in Fig. 15, in which a moving load is supposed to pass continuously over the span from the left-hand end A. It is required to determine what position of this uniform load will produce the greatest shear at the section C.

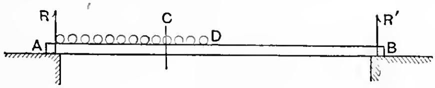


FIG. 15.

Let the moving load extend from A to any point D to the right of C. The two reactions R and R' may be found by the methods already indicated. Let W represent the uniform load resting on the portion CD of the span. The shear S' existing at C will be

$$S' = R' - W. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (21)$$

Let R''' be that part of R' which is due to W, and R'' that part due to the load on AC. Evidently R''' is less than W; then

$$S' = R'' + R''' - W. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (22)$$

Since the negative quantity W is greater than the positive quantity R''', S' will have its greatest value when both W and R''' are zero. Hence the greatest shear at the point C will exist when

$$S' = R''. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (23)$$

Obviously the loading must extend at least from A to C in order that R'' may have its maximum value. Hence *the greatest shear at any section will exist when the uniform load extends from the end of the span to that section, whatever may be the density of the load.*

If the segment of the span covered by the moving load is greater than one half the span, the maximum shear is called the *main shear;* but if that segment is less than one half the span, the maximum shear is called the *counter-shear.* The reason for these two names will be apparent later in the discussion of bridge-trusses.

This rule for determining the maximum shear at any section of a beam is equally applicable to bridge-trusses under certain conditions, and has an important bearing upon the determination of the greatest stresses in some of the members of bridge-frames, although it has less importance now than it had in the earlier days of bridge building.

85. Bending Moments and Shears for Cantilever Beams.—The case of a loaded overhanging beam or cantilever bracket, as shown in Fig. 16, is sometimes found. In that figure a single weight W is supposed to be applied at the end, while a uniform load w per unit of length extends over its length l. The bending moment at any point C distant x from the end will obviously be

$$M = Wx + \frac{wx^2}{2}. \quad \ldots \ldots \quad (24)$$

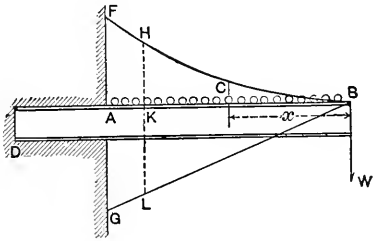


FIG. 16.

The greatest value of the bending moment will be found by placing x equal to l in equation (24), and it will have the value

$$M_1 = Wl + \frac{wl^2}{2}. \quad \ldots \ldots \quad (25)$$

The shear at any point and at the end A respectively will be

$$S = W + wx \quad \text{and} \quad S_1 = W + wl. \quad \ldots \quad (26)$$

The shear due to W is equal to itself and is constant throughout the whole length of the beam.

The second term of the second member of equation (24) is the equation of a parabola with its vertex at B, Fig. 16. Hence if AF be laid off equal to $\frac{wl^2}{2}$, and if the parabola FHB be drawn, any vertical intercept, as HK, between that curve and AB will represent the bending moment at the corresponding point. On the other hand, the first term of the second member of equation (24) shows that the bending moment due to W varies directly as the distance from B. Hence if AG be laid off vertically downward from A equal to Wl to any convenient scale, then any intercept, as KL, between AB and BG will represent the bending moment due to W at the corresponding point of the beam.

86. Greatest Bending Moment with any System of Loading.— One of the most important positions of loading to be established either for simple beams or for bridge-trusses is that at which any given system of loading whatever is to be placed on any span so as to produce the maximum bending moment at any prescribed point in that span. In order to make the case perfectly general a system of arbitrary loads, like that shown in Fig. 17, is assumed and the system is supposed to be a moving one.

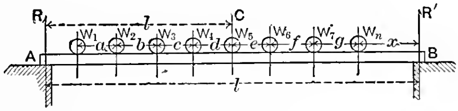


FIG. 17.

The separate loads are placed at fixed distances apart, indicated by the letters a, b, c, d, etc., W_1 being supposed to be at the head of the train, while W_n is the last load having a variable distance x between it and the end of the span. In Fig. 17 this system of moving loads or train is supposed to pass over the span l from right to left. The problem is to determine the position of the loading, so that the bending moment at the section C of the beam or truss will be a maximum, the section C being at the distance l' from the left-hand end of the span. The complete analysis of this problem is comparatively simple and may readily be found, but it is not necessary for the accomplishment of the present purpose to give it here. In order to exhibit the formula which expresses the desired condition, let $W_{n'}$ be that weight which is really placed at C, but which is assumed to be an indefinitely short distance to the left of that point, for a reason which will presently be explained. The equation of condition or criterion sought will then be the following:

$$\frac{l'}{l} = \frac{W_1 + W_2 + \ldots + W_{n'}}{W_1 + W_2 + W_3 + \ldots + W_n}. \quad . \quad . \quad . \quad . \quad (27)$$

If the loads are so placed as to fulfil the condition expressed in equation (27), the bending moment at section C will be a maximum. If the variation in the train weights is very great, it is possible that there may be more than one position of the train which will satisfy that equation. It is necessary, therefore, frequently to try different positions of the loading by that criterion and then ascertain which of the resulting maximum moments is the greatest. It is not usually necessary to make more than one or two such trials. The application of the equation is therefore simple and involves but little labor.

It will usually happen that $W_{n'}$ in equation (27) is not to be taken as the whole of that weight, but only so much of it as may be necessary to satisfy the equation. This is simply assuming that any weight, W, may be considered as made up of two separate weights placed indefinitely near to each other, which is permissible.

After having found the position of loading which satisfies equation (27), the resulting maximum bending moment will take the following form:

$$M_1 = \frac{l'}{l}[W_1 a + (W_1 + W_2)b + \ldots + (W_1 + W_2 + \ldots + W_n)x]$$
$$- W_1 a - (W_1 + W_2)b - \ldots - (W_1 + W_2 + \ldots + W_{n'-1})(?). \quad (28)$$

In this equation x corresponds to the position of loading for maximum bending, while the sign (?) represents the distance between the concentrations $W_{n'-1}$ and $W_{n'}$. This equation has a very formidable appearance, but its composition is simple and it is constantly used in making computations for the design of railroad bridges. The loads W_1, W_2, W_3, etc., represent the actual weights on the driving-axles and other axles of locomotives, tenders, and cars, and the spacings a, b, c, etc., are the actual spacings found between those axles. In other words, these quantities are the actual weights and dimensions of the different portions of moving railroad trains.

The computations indicated by equation (28) are not made anew in every instance. Concentrated weights of typical locomotives, tenders, and cars are prescribed by different railroad companies for their different classes of trains, ranging from the heaviest freight traffic to the lightest passenger train. A tabu-

CHAPTER VIII.

88. The Truss Element or Triangle of Bracing.—A number of the preceding formulæ find their applications to bridge-trusses, as well as to beams; hence it is necessary to give attention at least to some simple forms of those trusses.

The skeleton of every bridge-truss properly designed to carry its load is an assemblage of triangles. In other words, the truss element, i.e., the simplest possible truss, is the triangular frame, such as is shown in skeleton in Figs. 18 and 18*a*. These simple triangular frames are sometimes called the King-post Truss. The action of such a triangular frame in carrying a vertical load is extremely simple. In Fig. 18 let the weight W be suspended

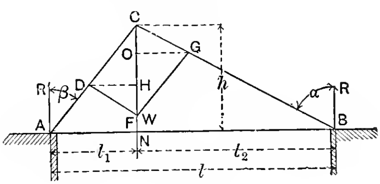


FIG. 18.

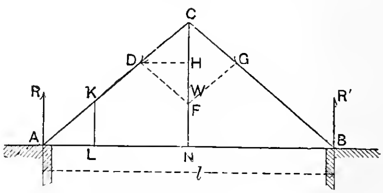


FIG. 18*a*.

from the apex C of the triangle. The line CF represents that weight, and if the latter be resolved into its two components parallel to the two upper members of the triangular frame, the two component forces CG and CD will result. If from D and G the horizontal lines DH and GO be drawn, those two lines will represent the horizontal components of the forces or stresses in the two bars CA and CB. The force HD will act to the left at the point A, and the force CG will act to the right at B, and as these two forces are equal and opposite to each other, equilibrium will result. Either of the horizontal forces will represent the magnitude of the tension in AB. Both AC and CB will be in compression, the former being compressed by the force CD, and the latter by the force CG. The manner of drawing a parallelogram of forces makes the triangle COG similar to CNB, and CHD similar to CNA; hence HW divided by CH will be equal to AN divided by NB. But HW is the vertical component of the stress in CB, while CH is the vertical component of the stress in AC, the latter being represented by the reaction R and the former by the reaction R'. It is seen, therefore, that the weight W is carried by the frame to the two abutment supports A and B, precisely as if it were a solid beam. In other words, the important principle is established that when weights rest upon a simple truss supported at each end they will produce reactions at the ends in accordance with the principle of the lever, precisely as in the case of a solid beam. In engineering parlance it is stated that the weight W is divided according to the principle of the lever, and that each portion travels to its proper abutment through the members of the triangular frame. If the two inclined members of the triangular frame are equally inclined to a vertical, the case of Fig. 18*a* results, in which one half of the weight goes to each abutment.

The triangular frame, with equally inclined sides, shown in Fig. 18*a*, is evidently the simplest form of roof-truss, constituting two equally inclined members with a horizontal tie.

89. Simple Trusses.—The simplest forms of trussing used for bridge purposes are those shown in Figs. 19, 20, and 21. There are many other forms which are exhibited in complete treatises on bridge structures, but these three are as simple as any, and they have been far more used than any other types. The horizontal members af and AB are called the "chords," the former being the upper chord and the latter the lower chord. The vertical and inclined members connecting the two chords are called the web members or braces. When a bridge is loaded, either by its own weight only, or by its own weight added to that of a moving train of cars, the upper chord will evidently be in compression, while the lower chord is in tension. A portion, which may be called a half, of the web members will be in tension and the other portion, or half, will be in compression.

The function of the upper and lower chords is to take up or resist the horizontal tension and compression which correspond to the direct stresses of tension and compression existing in the longitudinal fibres of a loaded solid or flanged beam. The metal designed to take these so-called direct stresses is concentrated in the chords of trusses, whereas it is distributed throughout the entire section of a beam, whether that beam be solid or flanged. The function of the web members of a truss is to resist the transverse or vertical shear which is represented by the algebraic sum of the reactions and loads. The total section of a solid beam resists these vertical shears, while the web only of a flanged beam is estimated to perform that duty. The horizontal shears, which have already been recognized as existing along the horizontal planes in a bent beam, are resisted by the inclined web members of a truss, the horizontal stress components being the horizontal shears, whereas the vertical shears are resisted by the vertical web members of a truss. If the web members are all inclined, as shown in Fig. 21, each web member resists both horizontal and vertical shear. It is thus seen that the members of a truss perform precisely the same duties as the various portions of either solid or flanged beams. Inasmuch as the chords of bridge-trusses resist the direct or horizontal stresses of tension and compression produced by the bending in the truss, it is obvious that the greatest chord stresses will be found at the centre of the span, and that they will be the smallest at the ends of the span. In the web members, on the contrary, since the vertical shear is the greatest at the ends of the span and equal to the reactions at those points, decreasing towards the centre precisely as in solid beams, the greatest web stresses will be found at the ends of the span and the least near the centre. It is obvious that the areas of cross-sections of either chords or web members must be proportioned to the stresses which they carry. Hence the distribution of stresses just described tends to a uniform distribution of the truss weights over the span.

90. The Pratt Truss Type.—In the discussion of these three simple types of trusses, the simplest possible loading of a perfectly uniform train will be assumed. The portions into which the trusses are divided by the vertical or inclined bracing are called panels. In Fig. 19, for instance, the points 1, 2, 3, 4, 5, and 6 of the lower chord and a, b, c, d, e, and f of the upper chord are called panel-points. The distance between each consecutive two of these points is called a panel length. The uniform train-load which is to be assumed will be represented by the weight W at each panel-point. This is called the "moving load" or "live load." The own weight of the structure is called the "dead load" or the "fixed load." The dead load per upper-chord panel will be taken as W', and W_1 for the lower chord. The loads to be used will, therefore, be as follows:

$$\begin{array}{ll} \text{Panel moving load} & =W; \\ \text{Upper-chord panel dead load} & =W'; \\ \text{Lower \quad `` \quad `` \quad `` \quad ``} & =W_1. \end{array}$$

There will also be used the length of panel and depth of truss as follows:

$$\begin{array}{ll} \text{Panel length} & =p; \\ \text{Depth of truss} & =d. \end{array}$$

In these simple trusses with horizontal upper and lower chords the stress in any inclined web members is equal to the shear multiplied by the secant of the inclination of the members to a vertical line. Also, at each panel-point every inclined web member, in passing from the end to the centre of the span, adds to either chord stress at that point an amount represented by the horizontal component of the stress which it carries; or, what is the same thing, an amount equal to the shear at the panel in question multiplied by the tangent of its angle of inclination to a vertical line.

It has already been shown in discussing solid beams that the greatest shear at any section will be found when the uniform moving load covers one of the segments of the span. This principle holds equally true for trusses carrying uniform panel-loads like those under consideration. In determining the stresses in these trusses, therefore, the inclined web members will take their greatest stresses when the moving train or load extends from the farthest end of the span up to the foot of the member in question. In this connection it is to be observed also that any two web members meeting in the chord which does not carry the moving load take their greatest stresses for the same position of the latter. The so-called "counter web members" take no stresses from the dead load.

Inasmuch as every load placed upon a truss will produce compression in the upper chord and tension in the lower, the greatest chord stresses will obviously exist when the moving load covers the entire span, and that condition of loading is to be used for the stresses in the following cases.

Bearing these general observations in mind, the ordinary simple method of truss analysis yields the tabulated statement of stresses given below for the three types selected for consideration. The first case to be treated is that of Fig. 19, which represents the Pratt truss type. The moving load is supposed to pass across the bridge from right to left. The plus sign indicates tension and the minus sign compression.

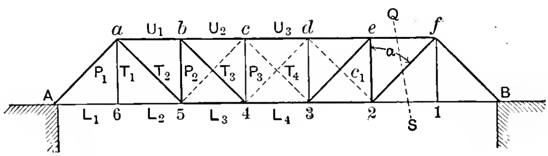


FIG. 19.

Stress in $c_1 = +\left(\frac{1}{7}+\frac{2}{7}\right)W \sec \alpha = \frac{3}{7}W \sec \alpha.$

Stress in $T_4 = +\left(\frac{1}{7}+\frac{2}{7}+\frac{3}{7}\right)W \sec \alpha = \frac{6}{7}W \sec \alpha;$

" " $T_3 = +\left[\left(\frac{1}{7}+\frac{2}{7}+\frac{3}{7}+\frac{4}{7}\right)W + W' + W_1\right] \sec \alpha$

$= \left(\frac{10}{7}W + W' + W_1\right) \sec \alpha;$

" " $T_2 = +\left[\left(\frac{1}{7}+\frac{2}{7}+\frac{3}{7}+\frac{4}{7}+\frac{5}{7}\right)W + 2w' + 2w_1\right] \sec \alpha$

$= \left(\frac{15}{7}W + 2w' + 2w_1\right) \sec \alpha;$

" " $T_1 = +(W + W_1).$

Stress in $P_3 = -\left(\frac{6}{7}W + W'\right);$

" " $P_2 = -\left(\frac{10}{7}W + 2W' + W_1\right);$

" " $P_1 = -3(W + W' + W_1) \sec \alpha.$

Stress in $L_1 =$ *Stress in* $L_2 = +3(W + W' + W_1) \tan \alpha;$

" " $L_3 =$ " " $L_2 \quad + 2(W + W' + W_1) \tan \alpha$

$= +5(W + W' + W_1) \tan \alpha;$

" " $L_4 =$ " " $L_3 \quad + (W + W' + W_1) \tan \alpha$

$= +6(W + W' + W_1) \tan \alpha.$

$$\begin{aligned}
&\textit{Stress in } U_1 = -\textit{Stress in } L_3 = -5(W+W'+W_1)\tan\alpha;\\
&\quad\text{``}\quad\text{``}\quad U_2 = -\quad\text{``}\quad\text{``}\quad L_4 = -6(W+W'+W_1)\tan\alpha;\\
&\quad\text{``}\quad\text{``}\quad U_3 = \quad\text{``}\quad\text{``}\quad U_2 = -6(W+W'+W_1)\tan\alpha.
\end{aligned}$$

It is easy to check any of the chord stresses by the method of moments. As an example, let moments first be taken about the panel-point 5 in the lower chord, and then about the panel-point c in the upper chord. The following expressions for the chord members U_1 and L_4 will be found, and it will be noticed that they are identical with the stresses for the same members given in the preceding tabulation, the counter-members, shown in broken lines, being omitted from consideration as they are not needed.

$$\textit{Stress in } U_1 = \frac{R.2p-(W+W'+W_1)p}{d}$$

$$= 5(W+W'+W_1)\frac{p}{d} = 5(W+W'+W_1)\tan\alpha. \qquad (29)$$

$$\textit{Stress in } L_4 = \frac{R.3p-2(W+W'+W_1).1\tfrac{1}{2}p}{d}$$

$$= 6(W+W'+W_1)\tan\alpha. \qquad (30)$$

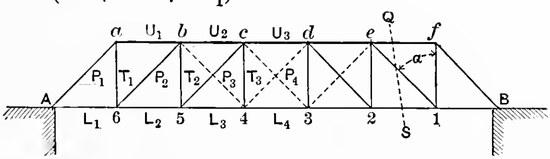


FIG. 20.

91. The Howe Truss Type.—The truss shown in Fig. 20 is the skeleton of the Howe truss, to which reference has already been made. The inclined web members are all in compression, while the vertical web members are all in tension. In the Howe truss all compression members are composed of timber. It has the disadvantage of subjecting the longest web members to compression. It thus makes the truss, if built all in iron or steel, heavier and more expensive than the trusses of the Pratt type. As in the preceding case, the moving train or load is supposed to pass across the bridge from B to A. Also, as before, the + sign indicates tension and the − sign compression. The greatest stresses, given in the tabulated statement below, can be computed or checked by the method of moments in this case, precisely as in the preceding.

Stress in $c_1 = -(\frac{1}{7}+\frac{2}{7})W \sec a = -\frac{3}{7}W \sec a.$
Stress in $P_4 = -(\frac{1}{7}+\frac{2}{7}+\frac{3}{7})W \sec a = -\frac{6}{7}W \sec a;$
" " $P_3 = -(\frac{10}{7}W + W' + W_1) \sec a;$
" " $P_2 = -(\frac{15}{7}W + 2W' + 2W_1) \sec a;$
" " $P_1 = -3(W + W' + W_1) \sec a.$

Stress in $T_3 = +(\frac{10}{7}W + W_1) \sec a;$
" " $T_2 = +(\frac{15}{7}W + W' + 2W_1) \sec a;$
" " $T_1 = +(3W + 2W' + 3W_1) \sec a.$

Stress in $L_1 = +3(W + W' + W_1) \tan a;$
" " $L_2 = +3(W + W' + W_1) \tan a + 2(W + W' + W_1) \tan a$
$= +5(W + W' + W_1) \tan a;$
" " $L_3 = +5(W + W' + W_1) \tan a + (W + W' + W_1) \tan a$
$= +6(W + W' + W_1) \tan a;$
" " $L_4 =$ *Stress in* L_3.

Stress in $U_1 = -$ *Stress in* L_1;
" " $U_2 = -$ " " L_2;
" " $U_3 = -$ " " L_3.

It will be noticed in the cases of Figs. 19 and 20 that upper and lower chord panels in the same lozenge or oblique panel have identically the same stresses, but with opposite signs. For instance, in Fig. 20 the stress in U_2 is equal in amount to that in L_2; and the same observation can be made in reference to the stresses in U_2 and L_4 of Fig. 19. This must necessarily always be the case in trusses having vertical web members.

In making computations for these forms of trusses it is very essential to observe where the first counter-member, as c_1, must be used. These counter-members may be omitted if the proper main web members near the centre of the span are designed to take both tension and compression.

92. **The Simple Triangular Truss.**—The truss shown in Fig. 21, in which all the web members have equal inclination to a vertical line, is sometimes called the Warren Truss, although that term has also been applied specially to this type of truss

so proportioned as to make the depth just equal to the panel length. As before, the moving train is supposed to pass over the bridge from B toward A, while the $+$ sign represents tension and the $-$ sign compression. The greatest stresses are the following.

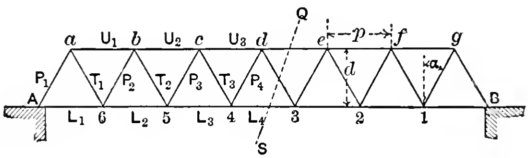


FIG. 21.

$$\textit{Stress in } P_4 = \begin{cases} -(\tfrac{6}{7}W + \tfrac{1}{2}W') \sec a, \text{ or} \\ +(\tfrac{6}{7}W - \tfrac{1}{2}W') \sec a; \end{cases}$$

$$\text{`` `` } P_3 = \begin{cases} -(\tfrac{10}{7}W + 1\tfrac{1}{2}W' + W_1) \sec a, \text{ or} \\ +(\tfrac{3}{7}W - 1\tfrac{1}{2}W' - W_1) \sec a; \end{cases}$$

$$\text{`` `` } P_2 = -(\tfrac{15}{7}W + 2\tfrac{1}{2}W' + 2W_1) \sec a;$$

$$\text{`` `` } P_1 = -(3W + 3\tfrac{1}{2}W' + 3W_1) \sec a.$$

$$\textit{Stress in } T_3 = \begin{cases} +(\tfrac{10}{7}W + \tfrac{1}{2}W' + W_1) \sec a, \text{ or} \\ -(\tfrac{3}{7}W - \tfrac{1}{2}W' - W_1) \sec a; \end{cases}$$

$$\text{`` `` } T_2 = +(\tfrac{15}{7}W + 1\tfrac{1}{2}W' + 2W_1) \sec a;$$

$$\text{`` `` } T_1 = +(3W + 2\tfrac{1}{2}W' + 3W_1) \sec a.$$

$$\textit{Stress in } L_1 = +3(W + W' + W_1) \tan a + \tfrac{1}{2}W' \tan a;$$

$$\begin{aligned} \text{`` `` } L_2 &= \textit{Stress in } L_1 + (5W + 5W' + 5W_1) \tan a \\ &= +8(W + W' + W_1) \tan a + \tfrac{1}{2}W' \tan a; \end{aligned}$$

$$\begin{aligned} \text{`` `` } L_3 &= \textit{Stress in } L_2 + 3(W + W' + W_1) \tan a \\ &= +11(W + W' + W_1) \tan a + \tfrac{1}{2}W' \tan a; \end{aligned}$$

$$\begin{aligned} \text{`` `` } L_4 &= \textit{Stress in } L_3 + (W + W' + W_1) \tan a \\ &= +12(W + W' + W_1) \tan a + \tfrac{1}{2}W' \tan a. \end{aligned}$$

$$\textit{Stress in } U_1 = -6(W + W' + W_1) \tan a;$$

$$\begin{aligned} \text{`` `` } U_2 &= -6(W + W' + W_1) \tan a - 4(W + W' + W_1) \tan a \\ &= -10(W + W' + W_1) \tan a; \end{aligned}$$

$$\begin{aligned} \text{`` `` } U_3 &= -10(W + W' + W_1) \tan a - 2(W + W' + W_1) \tan a \\ &= -12(W + W' + W_1) \tan a. \end{aligned}$$

The chord stresses may be checked or found by the method of moments, precisely as in the case of Fig. 19. If, for instance, it is desired to determine the stresses in the upper chord member

U_2, moments must be taken about the lower-chord panel-point 5, and about the upper-chord panel-point d for the lower-chord stress in L_4. Taking moments about those points, results given in equations (31) and (32) will at once follow, which it will be observed are identical with the values previously found for the same members.

$$\textit{Stress in } U_2 = -\frac{(3W + 3\tfrac{1}{2}W' + 3W_1) \,.\, 2p - 2W'p - (W + W_1)p}{d}$$

$$= -10(W + W' + W_1) \tan \alpha. \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \qquad (31)$$

$$\textit{Stress in } L_4 = +\frac{(3W + 3\tfrac{1}{2}W' + 3W_1).3\tfrac{1}{2}p - 3(W + W_1).1\tfrac{1}{2}p - 3W'.2p}{d}$$

$$= +12(W + W_1 + W') \tan \alpha + \tfrac{1}{2}W' \tan \alpha. \qquad . \quad . \qquad (32)$$

93. Through and Deck Bridges.—These simple trusses have all been taken as belonging to the "through" type, i.e., the moving load passes along their lower chords. It is quite common to have the moving load pass along the upper chords, in which cases the bridges are said to be "deck" structures. The general methods of computation are precisely the same whether the trusses be deck or through. It is only necessary carefully to observe that the application of the methods of analysis depends upon the position of each panel-load as it passes across the structure.

94. Multiple Systems of Triangulation.—Figs. 19, 20, and 21 exhibit what are called single systems of triangulation or single

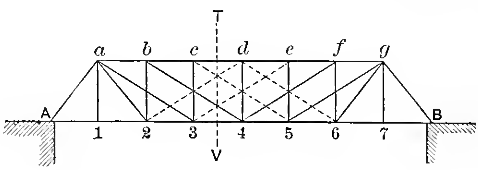


FIG. 22.

systems of bracing, but in each of those types the system of web members may be double or triple; in other words, they may be manifold. There have been many bridges built in which two or more systems of bracing are employed. Fig. 22 represents a truss with a double system of triangulation, known at one time as the Whipple truss. Fig. 23, again, exhibits a quadruple system of triangulation with all inclined web members. The

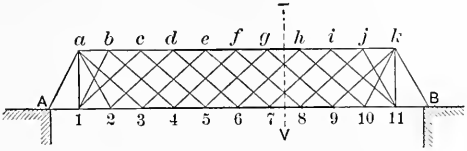


FIG. 23.

method of computation for such manifold systems is precisely the same as for a single system, each system in the compound truss being treated as carrying those loads only which rest at its panel-points. This procedure is not quite accurate. The complete consideration of an exact method of computation would take the treatment into a region of rather complicated analysis beyond the purposes of these lectures, but its outlines will be set forth on a later page. The exact method of treatment of two or more web systems involves the elastic properties of the material of which the trusses are composed. In the best modern bridge practice engineers prefer to design trusses of all lengths with single web systems, although the panels are frequently subdivided to avoid stringers and floor-beams of too great weight.

95. Influence of Mill and Shop Capacity on Length of Span.—In the early years of iron and steel bridge building the sizes of individual members were limited by the shop capacity for handling and manufacturing, and by the relatively small dimensions of bars of various shapes, and of plates which could be produced by rolling-mills. As both mill and shop processes have advanced and their capacities increased, corresponding progress has been made in bridge design. Civil engineers have availed themselves of those advances, so that at the present time single-system trusses with depths as great as 85 feet or more and spans of over 550 feet are not considered specially remarkable.

96. Trusses with Broken or Inclined Chords.—As the lengths of spans have increased certain substantial advantages have been gained in design by no longer making the upper chords horizontal in the case of long through-spans, or indeed in the cases of through-spans of moderate length. The greatest bending moments and the greatest chord stresses have been shown to exist at the centre of the span, while the greatest web stresses are found near the ends. Trusses may be lightened in view of those considerations by making their depths less at the ends than at the centre. This not only decreases the sectional areas of the heaviest web members near the ends of the truss, but also shortens them. It adds somewhat to the sectional area of the end upper-chord members, but the resultant effect is a decrease in total weight of material and increased stability against wind pressure by the decreased height and less exposure near the ends. It has therefore come to be the ruling practice at the present time to make through-trusses with inclined upper chords for practically all spans from about 200 feet upward. A skeleton diagram of such a truss is given in Fig. 24.

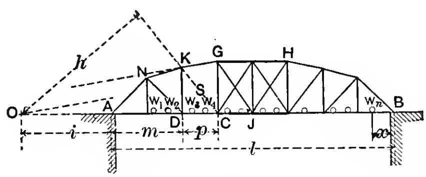


FIG. 24.

97. Position of any Moving Load for Greatest Web Stress.—In the preceding treatment of bridge-trusses with parallel and horizontal chords a moving or live load has been taken as a series of uniform weights concentrated at the panel-points. This simple procedure was formerly generally used, and at the present time it is occasionally employed, but it is now almost universal practice to assume for railroad bridges a moving load consisting of a series of concentrations, which represent both in amount and distribution the weights on the axles of an actual railroad train. If a bridge is supposed to be traversed by such a train, it becomes necessary to determine a method for ascertaining the positions of the train causing the greatest stresses in the various members of the bridge-truss. The mathematical demonstration of the formulæ determining those positions of loading need not be given here, but it can be found in almost any standard work on bridges.

In order to show concisely the results of such a demonstration let it be desired to find the position of a moving load which will give the greatest stress to any web member, as S in Fig. 24. Let the point of intersection of GK and DC be found in the point O, then let CK be extended, and on its extension let the perpendicular h be dropped from O. The distance of the point O from A, the end of the span, is i, while m is the distance AD. Using the same notation which has been employed in the discussion of beams, together with that shown in Fig. 24, equation (33) expresses the condition to be fulfilled by the train-loads in order that S shall have its greatest stress. The first parenthesis in the second member of that equation represents the load between the panel p and the left end of the span, while the second parenthesis represents the load in panel p itself.

$$W_1+W_2+\ \ldots\ +W_n=-\frac{l}{i}(W_1+W_2+\text{ etc.})$$

$$+(W_3+W_4+\text{etc.})\frac{l(m+i)}{pi}. \qquad (33)$$

It will be noticed in equation (33) that the quantity m shows in what panel the inclined web member whose greatest stress is desired is located, and it is important to observe that panel carefully. If, for instance, the vertical member KD were in question, the point O would be located at the intersection of the panel NK and the lower chord of the bridge. In other words, the point O must be at the intersection of the two chord members belonging to the same panel in which the web member is located.

98. Application of Criterions for both Chord and Web Stresses. —The criterion, equation (33), belongs to web members only. If it is desired to find the position of moving load which will give the greatest chord stresses in any panel, equation (27), already established for beams, is to be used precisely as it stands, the quantity l' representing the distance from one end of the span to the panel-point about which moments are taken.

If the desired positions of the moving load for greatest stresses have been found by equations (27) and (33), those stresses themselves are readily found by taking moments about panel-points for chord members and about the intersection-points O, Fig. 24, for web members. These operations are simple in character and are performed with great facility. Tabulations and diagrams are made for given systems of loading by which these computations are much shortened and which enable the numerical work of any special case to be performed quickly and with little liability to error. These tabulations and diagrams and other shortening processes may be found set forth in detail in many publications and works on bridge structures. They constitute a part of the office outfit of civil engineers engaged in structural work.

The criterion, equation (27), for the greatest bending moments in a bridge is applicable to any truss whatever, whether the chords are parallel or inclined, but it is not so with equation (33). If the chords of the trusses are parallel, the quantity i in equation (33) becomes infinitely great, and the equation takes the following form:

$$W_1+W_2+\ \ldots\ +W_n=\frac{l}{p}(W_3+W_4+\text{etc.}). \quad . \quad . \quad (34)$$

Ordinarily the span l divided by the panel length p is equal to the number of panels in the span. Hence equation (34) shows, in the case of parallel or horizontal chords, that when the moving load is placed for the greatest web stress in any panel, the total load on the bridge is equal to the load in that panel multiplied by the total number of panels.

99. Influence Lines.—A graphical method, known as that of "influence lines," is used for determining the greatest shears and bending moments caused by a train of concentrated weights passing along a beam or bridge-truss. Obviously it must express in essence that which has already been shown by the formulæ which determine positions of moving loads for the greatest shears and bending moments. In reality it is the application of graphical methods which have become so popular to the determination of the greatest stresses in beams and bridges.

100. Influence Lines for Moments both for Beams and Trusses. —It is convenient to construct these influence lines for an arbitrary load which may be considered a unit load; the effect of any other load will then be in proportion to its magnitude. The results determined from influence lines drawn for a load which may be considered a unit can, therefore, be made available for other loads by multiplying the former by the ratio between any desired load and that for which the influence lines are found.

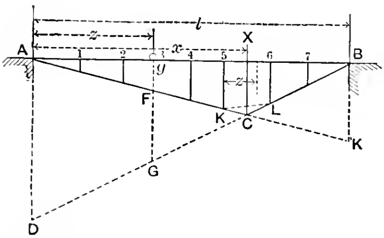


FIG. 25.—Bending Moment in a Simple Beam.

AB in Fig. 25 represents a beam simply supported at each end, so that any load g resting upon it will be divided between the points of support, according to the law of the lever. Let it be desired to determine the bending moment at the section X produced by the load g in all of its positions as it passes across the span from A to B. Two expressions for the bending moment must be written, one for the load g at any point in AX, and the other for the load at any point in BX. The expression for the first bending moment is

$$M = g\frac{z}{l}(l-x), \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (a)$$

and that for the latter

$$M' = g\frac{l-z}{l}x. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (b)$$

As shown in the figure, z and x, the latter locating the section at which the bending moments are to be found, are measured to the right from A. Equation (a) shows that if the quantity $g(l-x)$ be laid off, by any convenient scale, as BK at right angles to AB, XC will represent the moment M by the same scale when $x=z$ or when z has any value between 0 and x. Similarly will

AD be laid off at right angles to AB by the same scale as before, to represent gx. Then when $x=z$ the expression for M' will have the same value XC as before. Hence if the lines AC and CB be drawn as parts of AK and DB, any vertical intercept between AB and ACB will represent the bending at X produced by the load g when placed at the point from which the intercept is drawn. The lines AC and CB are the influence lines for the bending moments produced by the load g in its passage across the span AB. It is to be observed that the influence lines are continuous only when the positions of the moving load are consecutive. In case those positions are not consecutive the influence lines are polygonal in form.

If there are a number of loads g resting on the span at the same time, the total bending moments produced at X will be found by taking the sum of all the vertical intercepts between AB and ACB, drawn at the various points where those loads rest. The influence lines drawn for a single load, therefore, may be at once used for any number of loads.

The load g is considered as a unit load. If the vertical intercepts representing the bending moments by the scale used are themselves represented by y, and if W represent any load whatever, the general expression for the bending moment at X, produced by any system of loads, will be

$$\frac{1}{g}\Sigma Wy. \qquad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (c)$$

If this expression be written as a series, the general value of the bending moment will be the following:

$$M=\frac{1}{g}(W_1y_1+W_2y_2+W_3y_3+\text{etc.}). \quad . \quad . \quad . \quad . \quad (d)$$

The effect of a moving train upon the bending moment at any given section is thus easily made apparent by means of influence lines. It is obvious that there will be as many influence lines to be drawn as there are sections to be considered. In the case of a truss-bridge there will be such a section at every panel-point.

A slight modification of the preceding results is to be made when the loads are applied to the beam or truss at panel-points only.

In Fig. 25 let 1, 2, 3, 4, 5, 6, and 7 be panel-points at which loads are applied, and let the load g be located at the distance z' to the right of panel-point 5, also let the panel length be p. The reactions at 5 and 6 will then be $R_5 = g\frac{p-z'}{p}$ and $R_6 = g\frac{z'}{p}$. The reactions at A will then be $R = g\frac{l-z}{l}$. Hence the moment at any section X in the panel in question will be

$$M = Rx - R_5(z' - (z-x)) = g\left[\frac{l-x}{l}z - (z-z'+p-x)\frac{z'}{p}\right]. \quad . \quad (e)$$

Remembering that $z-z'$ is a constant quantity, it is at once clear that the preceding expression is the equation of a straight line, with M and z or z' the variables. If $z'=0$, equation (e) becomes identical with equation (a), while if $z'=p$, it becomes identical with equation (b). Hence the influence line for the panel in which the load is placed, as 5–6, is the straight line KL. It is manifest that when the load g is in any other panel than that in which the section X is located, the effect of the two reactions at the extremities of that panel will be precisely the same at the section as the weight itself acting along its own line of action. Hence the two portions AK and BL of the influence line are to be constructed as if the load were applied directly to the beam or truss, and in the manner already shown. The complete influence line will then be $AKLB$, and it shows that the existence of the panel slightly reduces the bending at any section within its limits. The panel 5–6, as treated, is that of a beam in which the bending moment will, in general, vary from point to point. If AB were a truss, however, X would always be taken at a panel-point, and no intercept between panel-points, as 5 and 6, would be considered.

101. Influence Lines for Shears both for Beams and Trusses.— The influence lines for shears in a simple beam, supported at each end, can be drawn in the manner shown in Fig. 25a. In that figure AB represents a non-continuous beam with span l supported at each end and a conventional load g at the distance z from A. The reaction at A will be

$$R=\frac{l-z}{l}g.$$

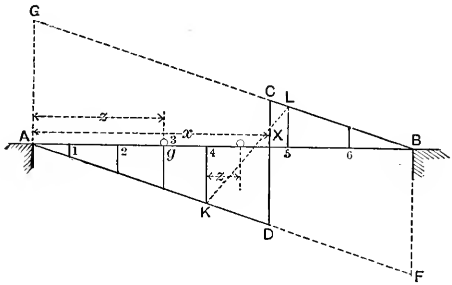


FIG. 25*a*.—Shear in a Simple Beam.

Let X be the section at which the shear for various positions of g is to be found. When g is placed at any point between A and X the shear S at the latter point will be

$$S=R=g=-g\frac{z}{l}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (f)$$

but when the load is placed between B and X the shear becomes

$$S'=R=g-g\frac{z}{l}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (h)$$

Obviously these two values of the shear are equations of two parallel straight lines, that represented by equation (f) passing through A, and that represented by equation (h) passing through B, the constant vertical distance between them being g. Hence let BF be laid off negatively downward and AG positively upward, each being equal to g by any convenient scale. The ordinates drawn from the various positions 1, 2, 3 . . . 6 of g on AB to AD and BC will be the shears at X produced by the load g at any point of the span, and determined by equations (f) and (h). The influence line, therefore, for the section X will be the broken line $ADCB$. When g is at X the sign of the shear changes, since the latter passes through a zero value.

If a train of weights W_1, W_2, W_3, etc., passes across the span, the total shear at X will be found by taking the sum of the vertical intercepts between AB and $ADCB$, drawn at the positions occupied by the various single weights of the train. If those single weights are expressed in terms of the unit load g, the shear S will have the value

$$S=\frac{1}{g}\Sigma Wy;$$

y being the general value of the intercept between AB and the influence line. The latter shows that the greatest negative shear at X will exist when the greatest possible amount of loading is placed on AX only, while the greatest positive shear at the same section will exist when BX only is loaded. If BX is the smaller segment of span, the latter shear is called the "counter-shear," and the former the "main shear."

If the loads are applied at panel-points of the span only, the treatment is the same in general character as that employed for bending moments. In Fig. 25a let 4 and 5 be the panel-points between which the load g is found, and let the panel length be p. Also, let z' be the distance of the weight g from panel-point 4. The reactions at A and 4 will then be

$$R=\frac{l-z}{l}g \quad \text{and} \quad R_4=\frac{p-z'}{p}g.$$

The shear at the section X for any position of the weight g will then be

$$S=R-R_4=g\left(\frac{z'}{p}-\frac{z}{l}\right). \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (k)$$

As this is the equation of a straight line, with S and z or z' for the coordinates, the influence line for the panel in which the section X is located will be the straight line represented by KL in Fig. 25a.

If z' is placed equal to 0 and p successively, then will equation (k) become identical with equations (f) and (h) in succession. The shears at points 4 and 5 will therefore take the same values as if the loads were applied directly to the beam. For the reasons stated in connection with the consideration of bending moments, loads in other panels than that containing the section for which the influence line is drawn will have the same effect on that section as if they were applied directly to the beam or truss. Hence $AKLB$ is the complete influence line for this case.

It is evident that there must be as many influence lines drawn as there are sections to be discussed. Also, if g is taken as some convenient unit, i.e., 1000 or 10,000 pounds, it is clear that the labors of computation will be much reduced.

102. Application of Influence-line Method to Trusses.—In considering both the bending moments and shears when the loads are applied at panel-points, it has been assumed, as would be the case in an ordinary beam, that the bending moments as well as the shears may vary in the panel; but this latter condition does not hold in a bridge-truss. Neither bending moment nor shear varies in any one panel. Yet the influence lines for moments and shears are to be drawn precisely as shown in Figs. 25 and 25*a*. The section X will always be found at a panel-point, and no intercept drawn within the limits of the panel adjacent to that section carrying the load g is to be used. This method will be illustrated by the aid of Fig. 25*b*.

The employment of influence lines may be illustrated by determining the moment and shear in a single section of the truss shown in Fig. 24, which is reproduced in Fig. 25*c*, when carrying the moving load exhibited in Fig. 25*b*, although its use may be much extended beyond this simple procedure.

The moving load shown in Fig. 25*b* is that of a railroad train consisting of a uniform train-load of 4000 pounds per linear foot drawn by two locomotives with the wheel concentrations shown; it is a train-load frequently used in the design of the heaviest class of railroad structures. If the criterion of equation (27) be applied to this moving load, passing along the truss shown in Fig. 25*c*, from left to right, it will be found that the greatest bending moment is produced at the section Q when the second driving-axle of the second locomotive is placed at the truss section in question, as shown in Fig. 25*c*.

The unit load to be used in connection with the influence lines will be taken at 10,000 pounds. Remembering that the panel lengths are each 30 feet, it will be seen that the panel-point Q is 150 feet from A. Hence the product gx will be 1,500,000 foot-pounds. Similarly the product $g(l-x)$ will be 900,000 foot-pounds. Laying off the first of these quantities, as AD, at a scale of 1,000,000 foot-pounds per linear inch, and the second quantity, as BK, by the same scale, the influence line ACB can at once be completed. Vertical lines are next to be drawn through the positions of the various weights, including one through the centre of the uniform train-load 110 feet in length resting on the truss. The vertical line through the centre of the uniform train-load is shown at O. By carefully scaling the vertical intercepts between AB and ACB, and remembering that each of the loads on the truss must be divided by 10,000, the following tabulated statement will be obtained, the sum of the intercepts for each set of equal weights being added into one item, and all the items of intercepts being multiplied by 1,000,000:

.195 ×	110	× .4 ×	1,000,000 =	8,580,000	foot-pounds.	
1.78 ×	2.6	×	" =	4,628,000	"	"
2.14 ×	4	×	" =	8,560,000	"	"
.485 ×	2	×	" =	970,000	"	"
1.525 ×	2.6	×	" =	3,965,000	"	"
.9 ×	4	×	" =	3,600,000	"	"
.12 ×	2	×	" =	240,000	"	"
			2)	30,543,000	"	"
Moment for one truss			=	15,271,500	"	"

The lever-arm of ef, i.e., the normal distance from Q to ef, is 39.7 feet. Hence the stress in ef is

$$\frac{15,271,500}{39.7} = 384,700 \text{ pounds.}$$

All the chord stresses can obviously be found in the same manner.

In order to place the same moving load so as to produce the greatest shear at the same section Q, the criterion of equation (33) must be employed. The dimensions of the truss shown in connection with Fig. 29 give the following data to be used in that equation: $i = 210$ feet, $m = 60$ feet, and $p = 30$ feet. Hence $\dfrac{l(m+i)}{pi} = 10\tfrac{2}{7}$, $\dfrac{l}{i} = 1\tfrac{1}{7}$. Introducing these quantities into equation (33), and remembering that the train moves on to the bridge from A, it would be found that the second axle of the first locomotive must be placed at the section Q, as shown in Fig. 25*d*, which exhibits the lower-chord panel-points numbered from 1 to 7. The conventional unit load g will be taken in this case at 20,000 pounds. It is represented as AG and BF (Fig.

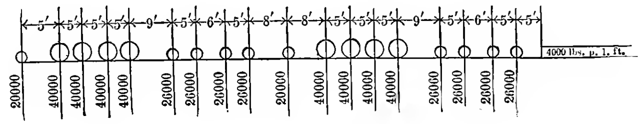


FIG. 25*b*.

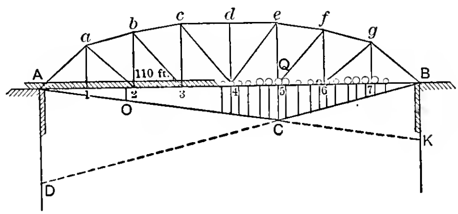


FIG. 25*c*.

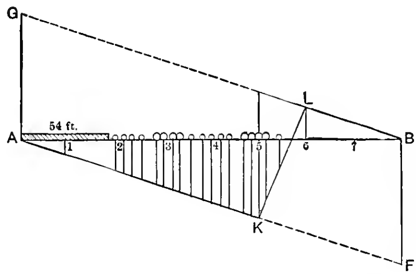


FIG. 25*d*.

25*d*), laid off at a scale of 10,000 pounds per inch. K is immediately under panel-point 5 and L is immediately above panel-point 6, hence the broken line $AKLB$ is the influence line desired. The vertical lines are then drawn from each train concentration in its proper position, all as shown, including the vertical line through the centre of the 54 feet of uniform train-load on the left. The summation of all the vertical intercepts between AB and the influence line AKL, having regard to the scale and to the ratio between the various loads and the unit load g, will give the following tabular statement:

.22 ×	54 ×	.2 ×	10,000	=	23,760	pounds.
2.2 ×		1.3 ×	"	=	28,600	"
3.02 ×		2 ×	"	=	60,400	"
.9 ×		1 ×	"	=	9,000	"
4.06 ×		1.3 ×	"	=	53,780	"
4.53 ×		2 ×	"	=	90,060	"
.5 ×		1 ×	"	=	5,000	"
					2)270,600	"
Shear for one truss				=	135,300	"

These simple operations illustrate the main principles of the method of influence lines from which numerous and useful extensions may be made.

CHAPTER IX.

103. Lateral Wind Pressure on Trusses.—The duties of a bridge structure are not confined entirely to the supporting of vertical loads. There are some horizontal or lateral loads of considerable magnitude which must be resisted; these are the wind loads resulting from wind pressure against both structure and moving train. In order to determine the magnitudes of these loads it is assumed in the first place that the direction of the wind is practically or exactly at right angles to the planes of the trusses and the sides of the cars. This assumption is essentially correct. There is probably nothing else so variable as both the direction and pressure of the wind. These variations are not so apparent in the exposure of our bodies to the wind, for the reason that we cannot readily appreciate even considerable changes either in direction or pressure. As a matter of fact suitable measuring apparatus shows that there is nothing steady or continued in connection with the wind unless it be its incessant variability. Its direction may be either horizontal or inclined, or even vertical, while within a few seconds its pressure may vary between wide limits. Under such circumstances the wind is as likely to blow directly against both bridge and train as in any other direction, and inasmuch as such a condition would subject the structure to its most severe duty against lateral forces, it is only safe and proper that the assumption should be made. The open work of bridge-trusses enables the wind to exert practically its full pressure against both trusses of a single-track bridge, or against even three trusses if they are used for a double-track structure. Hence it is customary to take the exposed surface of bridge-trusses as the total projected area on a plane throughout the bridge axis of both trusses if there are two, or of three trusses if there are three. Inasmuch as the floor of a bridge from its lowest point to the top of the rails or other highest point of the floor is practically closed against the passage of the wind, all that surface between the lowest point and the top of the rail or highest floor-member is considered area on which wind pressure may act.

Many experimental observations show that on large surfaces, greater perhaps than 400 or 500 square feet in area, the pressure of the wind seldom exceeds 20 or 25 pounds per square foot, while it may reach 80 or 90 pounds, or possibly more on small surfaces of from 2 to 40 or 50 square feet in area. This distinction between small and large exposed areas in the treatment of wind pressures is fundamental and should never be neglected.

This whole subject of wind pressures has not yet been brought into a completely definite or well-defined condition through lack of sufficient experimental observations, but in order to be at least reasonably safe civil engineers frequently, and perhaps usually, assume a wind pressure acting simultaneously on both bridge and train at 30 pounds per square foot of exposed surface and 50 pounds per square foot of the total exposed surface of a bridge structure which carries no moving load. This distinction arises chiefly from the fact that a wind pressure of 30 pounds per square foot on the side of many railroad trains, particularly light ones, will overturn them, and it would be useless to use a larger pressure for a loaded structure. There have been wind pressures in this country so great as to blow unloaded bridges off their piers; indeed in one case a locomotive was overturned which must have resisted a wind pressure on its exposed surface of not less than 90 pounds and possibly more than 100 pounds per square foot.

The consideration of wind pressure is of the greatest importance in connection with the high trusses of long spans, as well as in long suspension and cantilever bridges, and in the design of high viaducts, all of which structures receive lateral wind pressures of great magnitude.

Some engineers, instead of deducing the lateral wind loads from the area of the projected truss surfaces, specify a certain amount for each linear foot of span, as in "The General Specifications for Steel Railroad Bridges and Viaducts" by Mr. Theodore Cooper it is prescribed that a lateral force of 150 pounds for each foot of span shall be taken along the upper chords of through-bridges and the lower chords of deck-bridges for all spans up to 300 feet in length; and that for the same spans a lateral force of 450 pounds for each foot of span shall be taken for the lower chords of through-spans and the upper chords of deck-spans, 300 pounds of this to be treated as a moving load and as acting on a train of cars at a line $8\frac{5}{10}$ feet above the base of rail.

When the span exceeds 300 feet in length each of the above amounts of load per linear foot is to be increased by 10 pounds for each additional 30 feet of span.

Special wind-loadings and conditions under which they are to be used are also prescribed for viaducts.

These wind loads are resisted in the bridges on which they act by a truss formed between each two upper chords for the upper portion of the bridge, and between each two lower chords for the lower portion of the structure.

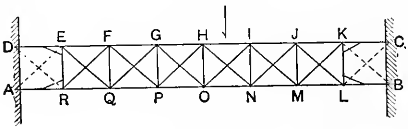


FIG. 26.

104. Upper and Lower Lateral Bracing.—Fig. 26 shows what are called the upper and lower lateral bracing for such trusses as are shown in the preceding figures. The wind is supposed to act in the direction shown by the arrow. *DERA* and *KLBC* are the two portals at the ends of the structure, braced so as to resist the lateral wind pressures. It will be observed that the systems of bracing between the chords make an ordinary truss, but in a horizontal plane, except in the case of inclined chords like that of Fig. 24. In the latter case the lateral trusses are obviously not in horizontal planes, but they may be considered in computations precisely as if they were. These lateral trusses are then treated with their horizontal panel wind loads just as the vertical trusses are treated for their corresponding vertical loads, and the resulting stresses are employed in designing web and chord members precisely as in vertical trusses. The wind stresses in the chords, in some cases, are to be added to those due to vertical loading, and in some cases subtracted. In other words, the resultant stresses are recognized and the chord members are so designed as properly to resist them. At the present time it is the tendency in the best structural work to make all the web members of these lateral trusses of such section that they can resist both tension and compression, as this contributes to the general stiffness of the structure. On account of the great variability of the wind pressures and the liability of the blows of greatest intensity to vary suddenly, some engineers regard all the wind load on structure or train as a moving load and make their computations accordingly. It is an excellent practice and is probably at least as close an approximation to actual wind effects as the assumption of a uniform wind pressure on a structure.

Both the lateral and transverse wind bracing of railroad bridges have other essential duties to perform than the resistance of lateral wind pressures. Rapidly moving railroad trains produce a swaying effect on a bridge, in consequence of unavoidable unevenness of tracks, lack of balance of locomotive driving-wheels, and other similar influences. These must be resisted wholly by the lateral and transverse bracing, and these results constitute an important part of the duties of that bracing. These peculiar demands, in connection with the lateral stability of bridges, make it the more desirable that the lateral and transverse bracing should be as stiff as practicable.

105. Bridge Plans and Shopwork.—After the computations for a bridge design are completed in a civil engineer's office they are placed in the drawing-room, where the most detailed and exact plans of every piece which enters the bridge are made. The numerical computations connected with this part of bridge construction are of a laborious nature and must be made with absolute accuracy, otherwise it would be quite impossible to put the bridge together in the field. The various quantities of bars, plates, angles, and other shapes required are then ordered from the rolling-mill by means of these plans or drawings. On receipt of the material at the shop the shopwork of manufacture is begun, and it involves a great variety of operations. The bridge-shop is filled with tools and engines of the heaviest description. Punches, lathes, planers, riveters, forges, boring and other machines of the largest dimensions are all brought to bear in the manufacture of the completed bridge.

106. Erection of Bridges.—When the shop operations are completed the bridge members are shipped to the site where the bridge is to be erected or put in place for final use. A timber staging, frequently of the heaviest timbers for large spans, called false works, is first erected in a temporary but very substantial manner. The top of this false work, or timber staging, is of such height that it will receive the steelwork of the bridge at exactly the right elevation. The bridge members are then brought onto the staging and each put in place and joined with pins and rivets. If the shopwork has not been done with mathematical accuracy, the bridge will not go together. On the accuracy of the shopwork, therefore, depends the possibility of properly fitting and joining the structure in its final position. The operations of the shop are so nicely disposed and so accurately performed that it is not an exaggeration to state that the serious misfit of a bridge member in American engineering practice at the present time is practically impossible. This leads to rapid erection so that the steelwork of a pin-connected railroad bridge 500 feet long can be put in place on the timber staging, or false works, and made safe in less than four days, although such a feat would have been considered impossible twenty years ago.

107. Statically Determinate Trusses. — The bridge structures which have been treated require but the simplest analysis, based only on statical equations of equilibrium of forces acting in one plane, i.e., the plane of the truss. It is known from the science of mechanics that the number of those equations is at most but three for any system of forces or loads, viz., two equations of forces and one of moments.

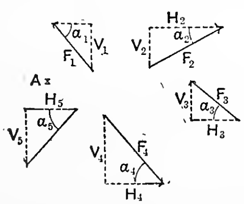


FIG. 27.

This may be simply illustrated by the system of forces F_1,

F_2, etc., in Fig. 27. Let each force be resolved into its vertical and horizontal components V and H. Also let l_1, l_2, etc. (not shown in the figure), be the normals or lever-arms dropped from any point A on the lines of action of the forces F_1, F_2, etc., so that the moments of the forces about that point will be F_1l_1, F_2l_2, etc. The conditions of purely statical equilibrium are expressed by the three general equations

$$H_1+H_2+\text{etc.}=F_1\cos\alpha_1+F_2\cos\alpha_2+\text{etc.}=0; \quad . \quad (35)$$
$$V_1+V_2+\text{etc.}=F_1\sin\alpha_1+F_2\sin\alpha_2+\text{etc.}=0; \quad . \quad . \quad (36)$$
$$Fl=F_1l_1+F_2l_2+\text{etc.}=0. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (37)$$

If all the forces except three are known, obviously those three can be found by the three preceding equations; but if more than three are unknown, those three equations are not sufficient to find them. Other equations must be available or the unknown forces cannot be found. In modern methods of stress determinations those other needed equations express known elastic relations or values, such as deflections or the work performed in stressing the different members of structures under loads. A few fundamental equations of these methods will be given.

In Figs. 19, 20, and 21 let the truss be cut or divided by the imaginary sections QS. Each section cuts but three members, and as the loads and reactions are known, the stresses in the cut members will yield but three unknown forces, which may be found by the three equations of equilibrium (35), (36), (37). If more than three members are cut, however, as in the section TV of Figs. 22 and 23, making more than three unknown equations to be found, other equations than the three of statical equilibrium must be available. Hence the general principle that *if it is possible to cut not more than three members by a section through the truss, it is statically determinate*, but *if it is not possible to cut less than four or more, the stresses are statically indeterminate.*

At each joint in the truss the stresses in the members meeting there constitute, with the external forces or loads acting at the same point, a system in equilibrium represented by the two equations (35) and (36). If there are m such joints in the entire structure, there will be $2m$ such equations by which the same number of unknown quantities may be found. Since equilibrium exists at every joint in the truss, the entire truss will be in equilibrium, and that is equivalent to the equilibrium of all the external forces acting on it. This latter condition is expressed by the three equations (35), (36), and (37), and they are essentially included in the number $2m$. Hence there will remain but $2m-3$ equations available for the determination of unknown stresses or external forces.

If, therefore, all the external forces (loads and reactions) are known, the $2m-3$ equations of static equilibrium can be applied to the determination of stresses in the bars of the truss or other structure. It follows, therefore, that the greatest number of bars that a statically determinate truss can have is

$$n=2m-3. \quad \quad \quad (38)$$

In Fig. 19 there are twelve joints and twenty-one members, omitting counter web members and the verticals *ab* and *fl*, which are, statically speaking, either superfluous or not really bars of the truss. Hence

$$m=12 \quad \text{and} \quad 2m-3=21. \quad \quad \quad (39)$$

Again, in Fig. 21 there are fifteen joints. Hence

$$m=15, \quad 2m-3=27,$$

and there are twenty-seven bars or members of the truss. The number of joints and bars in actual, statically determinate trusses, therefore, confirm the results.

108. Continuous Beams and Trusses—Theorem of Three Moments.—These considerations find direct application to what are known as "continuous beams," i.e., beams (or trusses) which reach continuously over two or more spans, as shown in Fig. 28.

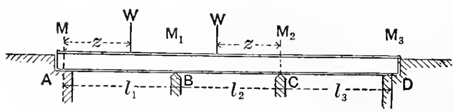


FIG. 28.

The beam shown is continuous over three spans, but a beam or truss may be continuous over any number of spans. In general the ends of the beam or girder may be fixed or held at the ends A and D, so that bending moments M and M_3 at the same points may have value. The bending moments at the other points of support are represented by M_1, M_2, etc. The points of support may or may not be at the same elevation, but they are usually assumed to be so in engineering practice. Finally, it is ordinarily assumed that the continuous structure is straight before being loaded, and that in that condition it simply touches the points of support. Whether the preceding assumptions are made or not, a perfectly general equation can be written expressing the relation between the bending moments over each set of three consecutive points of support, as M, M_1, and M_2, or M_1, M_2, and M_3. Such an equation expresses what is called the "Theorem of Three Moments." It is not necessary to give the most general form of this theorem, as that which is ordinarily used embodies the simplifying assumptions already described. This simplified form of the "Theorem of Three Moments" applied to the case of Fig. 28 will yield the following two equations:

$$Ml_1 + 2M_1(l_1 + l_2) + M_2l_2 + \frac{1}{l_1}\overset{1}{\Sigma} W(l_1^2 - z^2)z$$
$$+ \frac{1}{l_2}\overset{2}{\Sigma} W(l_2^2 - z^2)z = 0. \quad (40)$$

$$M_1l_2 + 2M_2(l_2 + l_3) + M_3l_3 + \frac{1}{l_2}\overset{2}{\Sigma} W(l_2^2 - z^2)$$
$$+ \frac{1}{l_3}\overset{3}{\Sigma} W(l_3^2 - z^2)z = 0. \quad (41)$$

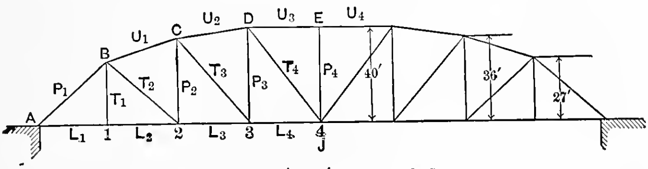
The figure over the sign of summation shows the span to which the summation belongs. If there is but one weight or load W in each span, the sign of summation is to be omitted. In an ordinary bridge structure or beam the ends are simply supported and $M = M_3 = 0$. In any case if the number of supports be n, there will be $n - 2$ equations like the preceding.

If the end moments M and M_3 are not zero, they will be determinable by the local conditions in each instance. In any event, therefore, they will be known, and there will be but $n - 2$ unknown moments to be found by the same number of equations. When the moments are known the reactions follow from simple formulæ.

109. Application to Draw- or Swing-bridges.—In general the reactions or supporting forces of the beams and trusses of ordinary civil-engineering practice are vertical, and all their points of application are known. Hence there are but two equations of equilibrium, equations (36) and (37), for external forces. These two equations for the external forces and the $n-2$ equations derived from the theorem of three moments are therefore always sufficient to determine the n reactions. After the reactions are known all the stresses in the bars or members of the trusses can at once be found. The preceding equations and methods as described are constantly employed in the design and construction of swing- or draw-bridges.

110. Special Method for Deflection of Trusses.—The method of finding the elastic deflections produced by the bending of solid beams has already been shown, but it is frequently necessary to determine the elastic deflections of bridge-trusses or other jointed or so-called articulate frames or structures. It is not practicable to use the same formulæ for the latter class of structures as for the former. The elastic deflection of a bridge- or roof-truss depends upon the stretching or compressions of its various members in consequence of the tensile or compressive forces to which they are subjected. Any method by which the deflection is found, therefore, must involve these elastic changes of length. There are a number of methods which give the desired expressions, but probably the simplest as well as the most elegant procedure is that which reaches the desired expression through the consideration of the work performed in the truss members in producing their elastic lengthenings and shortenings.

The general features of this method can readily be shown by reference to Fig. 29. It may be supposed that it is desired to find the deflection of any point, as J, of the lower chord produced both by the dead and live load which it carries. It is known from what has preceded that every member of the upper chord will be shortened and that every member of the lower chord will be lengthened; and also that generally the vertical web members will be shortened and the inclined web members lengthened. If there can be obtained an expression giving that part of the deflection of J which is due to the change of length of any one member of the truss independently of the others, then that expression may be applied to every other member in the entire truss, and by taking the sum of all those effects the desired deflection will at once result. While this expression will be found for some one particular truss member, it will be of such a general form that it may be used for any truss member whatever; it will be written for the upper-chord member BC in Fig. 29.



8 panels @ 30'=240' c. to c. end pins

FIG. 29.

The general problem is to determine the deflection of the point J when the bridge carries both dead and moving load over the entire span, as shown in Fig. 29. The general plan of procedure is first to find the stresses due to this combined load in every member of the truss, so that the corresponding lengthening or shortening is at once shown. The effect of this lengthening and shortening for any single member BC in producing deflection at J is then determined; the sum of all such effects for every member of the truss is next taken, and that sum is the deflection sought. In this case the vertical deflection will be found, because that is the deflection generally desired in connection with bridge structures, but precisely the same method and essentially the same formulæ are used to find the deflection in any direction whatever. The following notation will be employed:

Let w = deflection in inches at any panel-point or joint of the truss;

" P = any arbitrary load or weight supposed to be hung at the point where the deflection is desired and acting as if gradually applied. This may be taken as unity;

" Z = stress produced in any member of truss by P;

" S = stress produced in any member of truss by the combined dead and moving loads;

Let l = length in inches of any member of the truss in which Z or S is found;
" A = area of cross-section of same member in square inches;
" E = coefficient of elasticity.

S or Z may be either tension or compression, and the formulæ will be so expressed that tension will be made positive and compression negative.

The change of length of the chord member BC produced by a stress gradually increasing from zero to S is $\frac{S}{AE}l$. If it be supposed that BC is a spring of such stiffness that it will be compressed by the gradual application of Z exactly as much as the shortening of the actual member by the stress S, the deflection of the point 4 with the weight P hung from it, and due to that compression alone, will be precisely the same as that due to the actual shortening of BC by the combined dead and moving loads.

It is known by one of the elementary principles of mechanics that, since P acts along the direction of the vertical deflection w, the work performed by the weight P over that deflection is equal to the work performed by Z over the change of length l. Hence

$$\frac{l}{2}Pw = \frac{l}{2}Z\frac{Sl}{AE}, \quad \text{or}$$

$$w = \frac{Z}{P}\frac{Sl}{AE}. \qquad (42)$$

The quantity $Z \div P$ is the stress produced in the member by a unit load applied at the joint or point where the deflection is desired. Again, $S \div A$ is the stress per unit of area, i.e., intensity of stress, in the member considered by the actual dead and moving loads. For brevity let these be written

$$\frac{Z}{P} = z \quad \text{and} \quad \frac{S}{A} = s;$$

then

$$w = \frac{zsl}{E}. \qquad (43)$$

If the influence of every member of the truss is similarly expressed, the value of the total deflection produced by the dead and moving loads will be

$$w = \sum \frac{zsl}{E}. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (44)$$

The sign of summation Σ indicates that the summation is to extend over all the web and chord members of the truss.

111. Application of Method for Deflection to Triangular Frame.—Before applying those equations to the case of Fig. 29 it is best to consider a simpler case, i.e., that of the triangular frame shown in Fig. 18. The reactions are

$$R = \frac{l_2}{l} W \quad \text{and} \quad R' = \frac{l_1}{l} W. \quad . \quad . \quad . \quad . \quad . \quad (45)$$

The stresses in the various members are:

In CB, $S = \frac{l_1}{l} W \sec \alpha$.

" CA, $S = \frac{l_2}{l} W \sec \beta$.

" AB, $S = \frac{l_2}{l} W \tan \beta = \frac{l_1}{l} W \tan \alpha$.

Also: $CB = h \sec \alpha$; area of section $= A_1$.
$CA = h \sec \beta$; " " " $= A_2$.
$AB = l$; " " " $= A_3$.

In this instance it is simplest to take $P = W$. Equation (44) then gives

$$w = \left(\frac{l_1^2}{l^2} \frac{h \sec^3 \alpha}{A_1} + \frac{l_2^2}{l^2} \frac{h \sec^3 \beta}{A_2} + \frac{l_2^2}{l^2} \frac{l \tan^2 \beta}{A_3} \right) \frac{W}{E}. \quad . \quad (46)$$

Let it be supposed that

$l = 25$ feet $= 300$ inches;
$h = 8$ feet 4 inches $= 100$ inches;
$l_2 = 16$ feet 8 inches $= 200$ inches and $l_1 = 100$ inches;
$\tan \beta = 1$; $\sec \beta = 1.414$;
$\sec \alpha = 2.24$;
$W = 10{,}000$ pounds.

If the bars are all supposed to be of yellow-pine timber, there may be taken

$$E = 1{,}000{,}000 \text{ pounds};$$
$$A_1 = 10'' \times 12'' = 120 \text{ square inches};$$
$$A_2 = 10'' \times 10'' = 100 \text{ square inches};$$
$$A_3 = 10'' \times 12'' = 120 \text{ square inches}.$$

The insertion of these quantities in equation (46) gives the deflection

$$w = .01042 + .01253 + .01111 = 0''.034. \quad . \quad . \quad . \quad (47)$$

Equation (47) is so written as to show the portion of the deflection due to each member of the frame.

In applying either equation (43) or equation (44) care must be taken to give each stress and its corresponding strain (lengthening or shortening) the proper sign. As the formulæ have been written and used, a tensile stress and its resulting stretch must each be written positive, while a compressive stress must be written negative. This holds true for both the stresses Z and S (or z and s). The magnitude of the assumed load P is a matter of indifference, since the stress Z will always be proportional to it and the ratio $P \div Z$ will therefore be constant. P is frequently taken as unity; or, as in the case just given, it may have any value that the conditions of the problem make most convenient.

112. Application of Method for Deflection to Truss.—In making application of the deflection formulæ to any steel railroad truss similar to that shown in Fig. 29, it will first be necessary to determine the stresses in all its members due to the dead and moving loads, since the deflection under the moving load is sought. These loads will be considered uniform, and that is sufficiently accurate for any railroad bridge. The moving train-load will be taken as covering the entire span, assumed, for a single-track railroad, 240 feet in length between centres of end pins. There are eight panels of 30 feet each, and the depth of truss at centre is 40 feet. Other truss dimensions are as shown in Fig. 29. The dead loads, or own weight, are taken at 400 pounds per linear foot of span for the rails and other pieces that constitute the track; at 400 pounds per linear foot for the steel floor-beams and stringers, and 1600 pounds per linear foot for the weight of trusses and bracing. The moving train-load will be taken at 4000 pounds per linear foot. This will make the panel-loads for each truss as follows:

Lower-chord dead load,	$30 \times 800 =$	24,000	pounds per panel.
Lower-chord moving load,	$30 \times 2000 =$	60,000	" " "
Total load on lower chord	$=$	84,000	" " "
Upper-chord dead load,	$30 \times 400 =$	12,000	" " "

The structure is a "through" bridge, hence all moving loads rest on the lower chord.

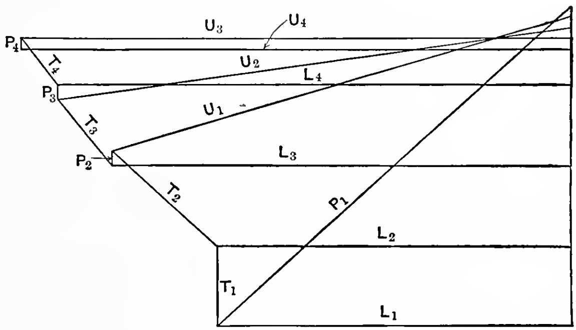


Fig. 30.

The stresses in the truss members due to the combined uniform dead and moving load are best found by the graphical method. One diagram only is needed to determine all the stresses, and it is shown in Fig. 30. This diagram is drawn accurately to scale, and the stresses measured from it are shown in the table on page 136.

The stresses in all the truss members due to the unit load hung at *J* are readily found by the single diagram shown in Fig. 31, also carefully drawn to scale. These stresses measured from the diagram are given in the table as indicated by the column z; they are also represented in equation (44) by the letter z. The quantity s in equation (44) is the intensity of the stress (pounds per square inch of cross-section of member) produced by the combined dead and moving loads in each member. As shown,

	S	*s*	*z*	*l*	*w*
L_1	+373,300	+12,000	+.555	360	+.08563
L_2	+373,300	+12,000	+.555	360	+.08563
L_3	+480,000	+12,000	+.833	360	+.1284
L_4	+540,000	+12,000	+1.125	360	+.1736
P_1	−502,300	−9,000	−.748	472	+.1132
U_1	−501,000	−9,500	−.870	376	+.1108
U_2	−544,800	−10,000	−1.135	363	+.1472
U_3	−576,000	−10,000	−1.50	360	+ 1928
T_1	+84,000	+9,000	0	324	....
T_2	+143,500	+10,000	+.3738	472	+.0629
P_2	−12,000	−1,000	−.250	432	+.00386
T_3	+93,720	+7,400	+.456	562	+.0677
P_3	+12,000	+1,000	−.35	480	−.0060
T_4	+60,000	+4,800	+.625	600	+.0643
P_4	−12,000	−1,000	0	480	....

Deflection for $\frac{1}{2}$ truss members $= 1.2300$ inches.
Deflection at $J = 2 \times 1.2300 = 2.4600$ inches.

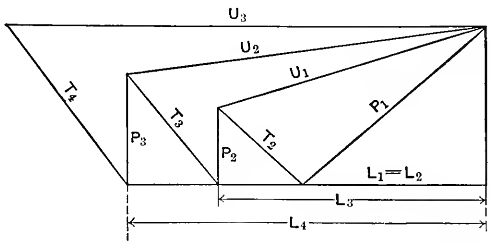


FIG. 31.

these stresses are least in the web members near the centre of the span, and greatest in the chord members. The lengths in inches of the truss members are shown in the proper column of the table. It will be observed that all counter web members are omitted, as they are not needed for the uniform load. The coefficient of elasticity (E) is taken at 28,000,000 pounds. The quantities represented by the second member of equation (44) are computed from these data, and they appear in the last column of the table, the sum of which gives the desired deflection in inches. The elements of the table show how much of the deflection is due to the chords and to the web members, and they show that disregarding the latter would lead to a considerable error.

As the deflection is usually desired in inches, the lengths of members must be taken in the same unit.